Investigation of Frequency-Modulation Signal Interference*

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Summary—The cause and mechanism of interference between two frequency-modulation signals are analyzed. It is shown that, while the interference of two frequency-modulation signals on the same channel is practically independent of receiver design, off-channel interference depends on the shape of discriminator curve beyond 120 kilocycles off resonance.

Methods are developed to calculate the amount of interference for a given receiver, in terms of the relative strength of the interfering signal. Receiver design modifications, which will reduce the amount of interference from different channels, are indicated.

Introduction

INTERFERENCE between several frequency-modulation signals operating on, or near, the same frequency is becoming an important receiver design problem with the steadily increasing number of transmitters being put into operation.

The object of this work is to present a quantitative analysis of the interference under conditions likely to be encountered in a frequency-modulation receiver, particularly in regard to the action of the discriminator. This phase of a frequency-modulation receiver has been neglected by most writers on the general subject, probably because of the widespread belief that the shape of the discriminator curve beyond the limits of modulation is of no consequence provided the limiter is operating perfectly. Methods are developed for analyzing and evaluating the effect of the shape of the discriminator curve upon the susceptibility to interference. Also, some of the possible remedies are outlined.

Interference from other frequency-modulation stations may be divided into two classifications: interference arising from stations on the same channel, and that from stations on different channels. These cases are treated separately, as follows: (1) co-channel interference, and (2) adjacent-and alternate-(second-) channel interference.

Definitions of the Symbols

F_c = the carrier frequency to which the receiver is tuned (desired signal)

F_n = the carrier frequency of the interfering signal

$F_{cn} = (F_c - F_n)$ = the beat frequency between the desired and interfering signals

F_m = modulating frequency

$\rho = c/n$ = the ratio of the amplitudes of the desired to interfering signals at the output of intermediate frequency

F_d = maximum frequency deviation due to frequency modulation ($F_d = 75$ kilocycles)

$\beta = F_d/F_m$

F_i = the frequency deviation (modulation due to the interfering signal)

S = output of desired signal corresponding to 75-kilocycle deviation (100 per cent modulation)

S/N = the signal-to-interference ratio in the audio

N = the output of the interfering signal

$W_c = 2\pi F_c$

$W_n = 2\pi F_n$

$W_{cn} = 2\pi F_{cn} = W_c - W_n$

$p = 2\pi F_m$

A = the amplitude.

General Theory

The two signals, desired and interfering, reach the limiter to form a composite signal, both frequency- and amplitude-modulated. This modulation has a fundamental frequency corresponding to the frequency difference between the two signals and in certain cases is high in harmonic content.

An amplitude limiter with ideal characteristics is used throughout this analysis. The expression for the composite signal at the output of such a limiter has been derived many times,[1] and so the derivation will not be given here. The expression is:

$$e = A \sin \left\{ \omega_c t + \beta \cos pt + \tan^{-1} \frac{\sin [\omega_{cn} t - \phi(t) + \beta \cos pt]}{\rho + \cos [\omega_{cn} t - \phi(t) + \beta \cos pt]} \right\}. \quad (1)$$

This is the most general form, when both signals are frequency modulated. The expression $\beta \cos pt$ is due to the desired signal modulation. The term $\phi(t)$ is due to the interfering signal modulation.

Let

$$\tan^{-1} \frac{\sin [\omega_{cn} t - \phi(t) + \beta \cos pt]}{\rho + \cos [\omega_{cn} t - \phi(t) + \beta \cos pt]} = \alpha. \quad (1a)$$

Frequency deviation due to the interfering signal can be found from

$$F_i = \frac{1}{2\pi} \frac{d\alpha}{dt} = \frac{F_{cn} - \frac{d\phi}{dt} - F_d \sin pt}{\frac{\rho + \cos [\omega_{cn} t - \phi(t) + \beta \cos pt]}{\frac{1}{\rho} + \cos [\omega_{cn} t - \phi(t) + \beta \cos pt]} + 1} \quad (2)$$

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[1] M. G. Crosby, "Frequency modulation noise characteristics," Proc. I.R.E., vol. 25, pp. 472–514; April, 1937.

When both signals are unmodulated carriers, $\phi(t)$, $\beta \cos pt$, and their derivatives vanish, and (2) becomes

$$F_i = \frac{F_{cn}}{\dfrac{\rho + \cos \omega_{cn} t}{\dfrac{1}{\rho} + \cos \omega_{cn} t} + 1}. \tag{2a}$$

This equation shows that F_i is not symmetrical; its two peaks can be obtained by substituting $\cos \omega_{cn} t \pm 1$.

$$\text{First peak} \quad F_{i_1} = \frac{F_{cn}}{\rho + 1}; \qquad (\cos \omega_{cn} t = 1) \tag{3}$$

$$\text{Second peak} \quad F_{i_2} = \frac{F_{cn}}{\rho - 1}; \qquad (\cos \omega_{cn} t = -1). \tag{4}$$

The asymmetry increases as ρ decreases. Expanded, (2a) becomes

$$F_i = F_{cn} \sum_{r=1}^{r=\infty} (-1)^{r+1} \frac{\cos r\omega_{cn} t}{\rho^r}. \tag{5}$$

There is no constant term in this expression: although nonsymmetrical, F_i has an average value equal to zero. Consequently, there is no frequency shift.

Co-Channel Interference

When the desired and interfering signals are on the same channel and the difference in frequency is within the audio range, it has been found experimentally that interference is strongest when both signals are unmodulated carriers. This case will be examined first.

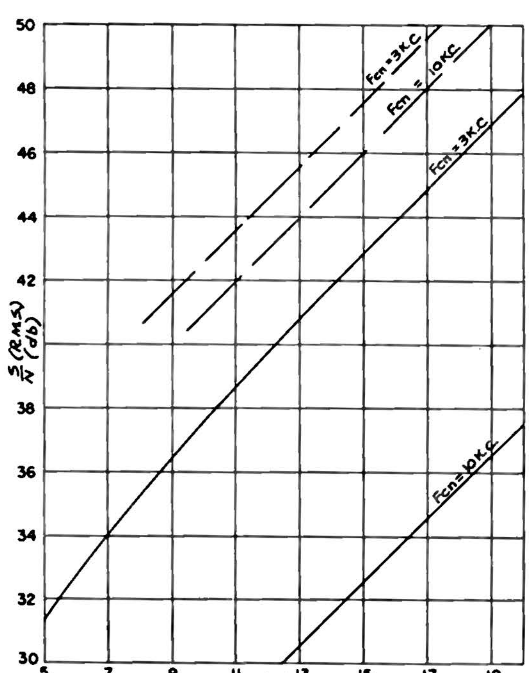

Fig. 1—Signal-to-interference ratio in output due to interfering signal on the same channel (both signals unmodulated) for two beat frequencies (F_{cn}).
——— Without de-emphasis
– – – – With de-emphasis.

(a) The root-mean-square value of the interference N can be found from (5).

$$N_{rms} = \frac{F_{cn}}{\sqrt{2}} \sqrt{\sum_{r=1}^{r=r_0} \frac{1}{\rho^{2r}}}$$

where

$$r_0 \leqq \frac{15}{F_{cn}} \quad (F_{cn} \text{ in kilocycles}). \tag{6}$$

The results were plotted as S/N (in decibels) against ρ, for different values of F_{cn} (Fig. 1). These curves were plotted for two cases: with no de-emphasis and with 75-microsecond de-emphasis. Without de-emphasis the total interference is proportional to F_{cn}, while with 75-microsecond de-emphasis it increases much more slowly with F_{cn} and for higher F_{cn} approaches asymptotically a constant value.

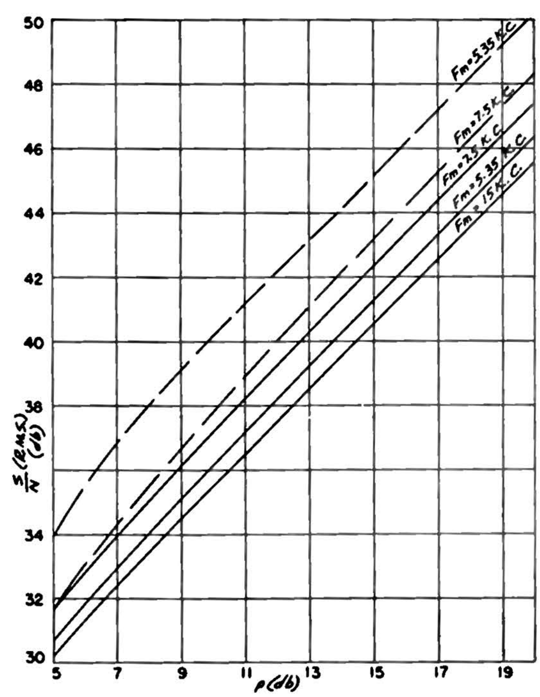

Fig. 2—Signal-to-interference ratio when both signals are on the same channel (one signal modulated)
——— $F_{cn} = 3$ kilocycles
– – – – $F_{cn} = 10$ kilocycles.

(b) When only one of the signals is frequency modulated, $\phi(t)$ in (2) vanishes and it becomes

$$F_i = \frac{F_{cn} - F_d \sin pt}{\dfrac{\rho + \cos[\omega_{cn} t + \beta \cos pt]}{\dfrac{1}{\rho} + \cos[\omega_{cn} t + \beta \cos pt]} + 1}. \tag{7}$$

This equation is expanded by binomial formula, then using sine and cosine addition formulas, the Bessel Functions expansion and recurrence formula, whereupon it is brought to a more convenient form:

$$F_i=\sum_{n=1}^{n=\infty}\sum_{r=0}^{r=\infty}\left\{J_r(n\beta)\left[(F_{cn}+rF_m)\cos\left(n\omega_{cn}t+rpt+\frac{\pi r}{2}\right)\right.\right.$$
$$\left.\left.+(F_{cn}-rF_m)\cos\left(n\omega_{cn}t-rpt+\frac{\pi r}{2}\right)\right]\right\}. \quad (8)$$

The root-mean-square values of N were calculated from this formula for different values of F_{cn} and F_m and were plotted against ρ (Fig. 2). In calculating the root-mean-square values only the components of audio frequencies (less than 15 kilocycles) were taken into consideration.

Equation (8) shows that the components of higher frequencies have greater amplitudes. Consequently, in this case, interference is much more affected by the de-emphasis than in case of two unmodulated carriers, because the higher frequencies are attenuated much more. Without de-emphasis, interference would be almost the same in both cases with and without modulation. With de-emphasis, however, interference will be considerably weaker when modulation is present.

Adjacent-Channel Interference

According to present Federal Communications Commission standards, 200-kilocycle adjacent-channel spacing is the closest that needs to be considered above that of 4-kilocycle common channel maximum spacing. After that, 400-kilocycle spacing will be considered. While the analysis is identical for both of these cases, the evaluation of the magnitudes to be expected in a practical receiver is different.

Suppose a frequency-modulation receiver is tuned to a certain signal of a frequency F_c (desired signal). There is another signal on the adjacent ($F_{cn}=200$ kilocycles) channel which interferes with the first. Let us assume that, at first, both signals are unmodulated carriers. Then F_{cn} and the harmonics will be far above the audio range, so there will be no audio interference.

If, then, the interfering signal is modulated 100 per cent, its frequency will swing 75 kilocycles in each direction. F_{cn} will vary with the modulation, from 125 to 275 kilocycles. The selectivity curve of an ordinary receiver is very steep between 125 and 275 kilocycles from resonance, and the gain of the receiver changes greatly between those frequencies. Thus, the amplitude of the interfering signal at the last intermediate-frequency circuit will change together with the frequency variation, and the signal will become amplitude, in addition to frequency, modulated at the output of the intermediate-frequency amplifier. The amount of this amplitude modulation depends upon the shape of the selectivity curve. With a typical selectivity curve of a present commercial receiver, amplitude modulation reaches 100 per cent at a frequency deviation between 35 and 45 kilocycles.

When the interfering signal is both frequency and amplitude modulated, ρ is no longer a constant, but becomes a function of time. When modulation is sinusoidal

$$\rho=\frac{c}{m(1+\cos pt)}\,.$$

If we substitute this expression into (1a) and get

$$F_i=\frac{1}{2\pi}\frac{d\alpha}{dt},$$

we find that all the terms that contain functions of F_m (modulating frequency) cancel out, only the terms that contain functions of beat frequency and its harmonics remaining in the final expression. As long as the discriminator curve is linear, frequency variation F_i will be transformed into corresponding amplitude modulation without distortion. Therefore, the amplitude variations at the output of the discriminator will not contain any modulating frequencies, and there will be no audible effect of the interference.

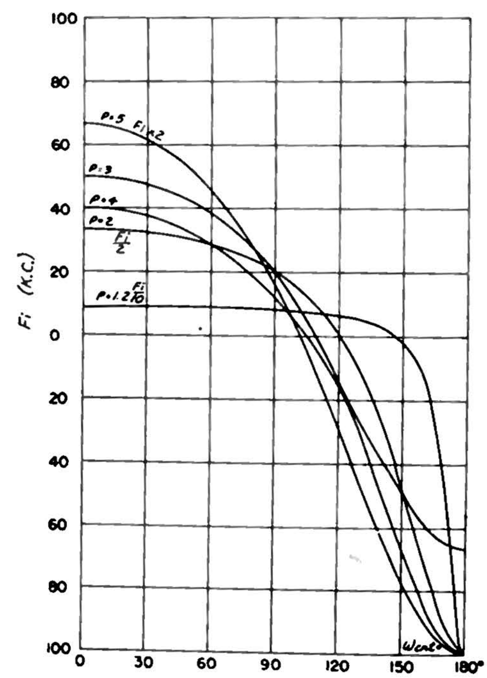

Fig. 3—Frequency deviation of the desired signal produced by interfering signals of different strengths ($F_{cn}=200$ kilocycles).

In order to determine the effect of the nonlinear part of the discriminator curve on the interference, we shall once more examine (3) and (4). For low ρ, F_i is not symmetrical; one of its peaks is much greater than the other. As was shown earlier (5), the average value of F_i is zero. As long as F_i is changed into amplitude variation A without distortion, the average value of A will also be zero, and so there will be no direct-current component in the output of the discriminator. However, as soon as one of the peaks extends into the nonlinear part of the discriminator curve, distortion is introduced, the average value of the discriminator output is no longer zero, and a direct-current component appears. For a fixed discriminator curve, the magnitude of this component depends on both F_{cn} and ρ. As was shown before, when the interfering signal is frequency modulated, it also becomes amplitude modulated. Then ρ and F_{cn} both vary with F_m, but ρ varies much faster (due to steepness of the selectivity curve). Since the magnitude of the direct-current component depends upon ρ, it will vary with ρ and consequently with F_m, thus introducing an audible interference.

The easiest method to find the magnitude of the direct-current component is graphical. First F_i (2a) is plotted for a half cycle against $W_{cn}t$ (Fig. 3). Then the values of F_i for different $W_{cn}t$ are applied to the discriminator curve[2] and the corresponding values of the amplitude are found. These amplitudes are plotted against $W_{cn}t$ and represent a half cycle of the discriminator output (Fig. 3). The direct-current component is then found from Fig. 4:

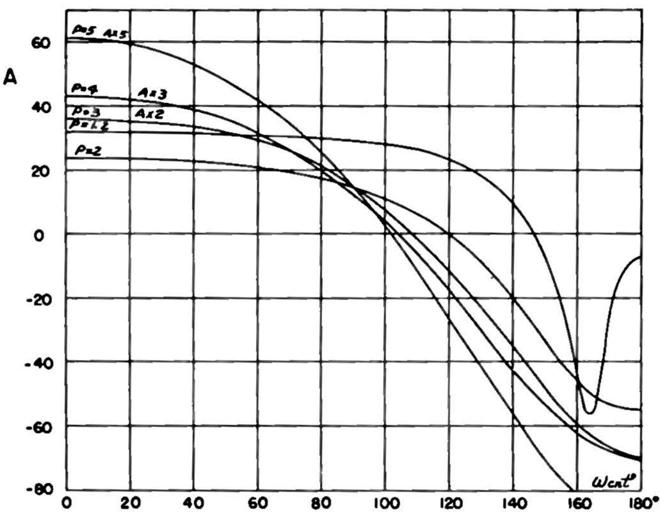

Fig. 4—Amplitude variations resulting from frequency deviation due to the interfering signals of different strength (Fig. 3).

$$\text{direct current} = \frac{\sum \text{area}}{\text{abscissa}}.$$

The actual amount of interference would depend on variation of ρ, and therefore on the particular selectivity curve used. It was found, however, that with the selectivity curves of the commercial receiver available, the actual interference with F_{cn} both 200 and 400 kilocycles was very close to that caused by the direct-current component 100 per cent modulated. The results calculated on this assumption were plotted as S/N for adjacent channel (Fig. 5).

In the case where the actual selectivity curve is available, the procedure to find the variation of direct-current component would be as follows: Three direct-current curves are plotted against ρ, for F_n and $F_{cn} \pm 75$ kilocycles. The variation of ρ corresponding to ± 75-kilocycle deviation of the interfering signal is found from the selectivity curve. Knowing how much ρ changes between $F_{cn}+75$ and $F_{cn}-75$ kilocycles, the exact variation of the direct-current component is then found from the three curves.

Although the above-described method is comparatively simple and easy to visualize, it is accurate only when the interfering-signal frequency falls between the peaks of the discriminator curve. In other cases a general method of sideband components must be used.

The general method is to expand expression (1) (simplified for the case of two unmodulated carriers). After expansion it takes a form:

$$e = \alpha_0 + \sin \omega_c t + \sum_{r=1}^{r=\infty} (-1)^r [b_r \sin (\omega_c - r\omega_{cn})t - a_r \sin (\omega_c + r\omega_{cn})t]. \quad (9)$$

(The derivation of this expression and the coefficients a_r and b_r is shown in the appendix.) The absolute values of the coefficients a_r, b_r were plotted against ρ (Fig. 6).

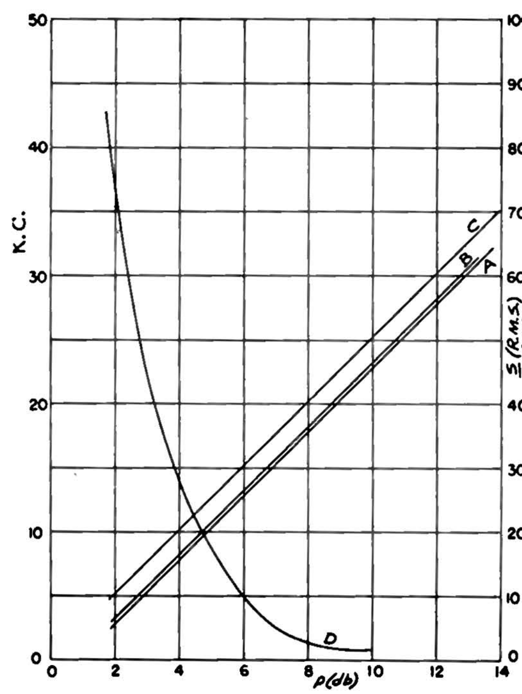

Fig. 5—Curves A, B, and C show signal-to-interference ratio in output due to interfering signals from adjacent and alternate channels.
A—F_n = 200 kilocycles (calculated by equivalent-frequency-deviation method).
B—F_{cn} = 200 kilocycles (sideband-components method).
C—F_{cn} = 400 kilocycles (sideband-components method).
D—Detuning (in kilocycles) required to reduce interference from signal in adjacent channel.

When signal e is applied to each diode circuit of a balanced discriminator, there is a direct-current component in the output of each diode. The difference of these components from two diodes is the resultant direct-current component D in the output of the balanced discriminator. $D = D_1 - D_2$ where D_1 and D_2 are the direct-current components in the output of each diode. The method to calculate D_1 and D_2 is shown in the appendix. As was shown before, when the interfering signal becomes frequency modulated, D changes with F_m and thus introduces an audible interference. From

[2] Discriminator curve taken from *RCA Report* LB-326, "Automatic-frequency control," Fig. 4, page 9 ($Q=25$).

(9) it can be seen that the sideband components have frequencies $F_c \pm rF_{cn}$, $r-1$, 2, 3. Fig. 6 shows that symmetrical components (those having the same value of r) have unequal amplitudes. When the discriminator curve is a straight line, the coefficients a_r, b_r (9) are such that $D_1 = D_2$, that is, $D=0$, and there is no interference. If the discriminator curve deviates from a straight line, D

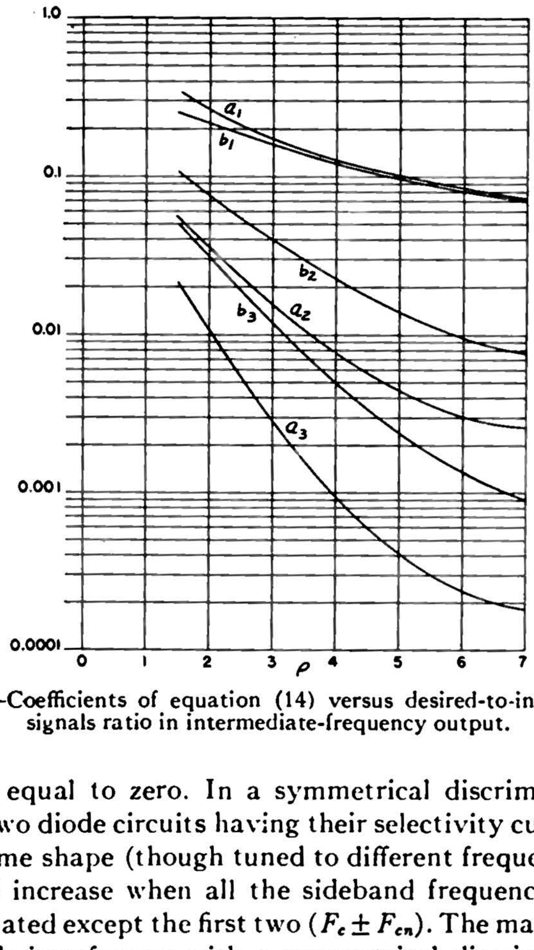

Fig. 6—Coefficients of equation (14) versus desired-to-interfering signals ratio in intermediate-frequency output.

is not equal to zero. In a symmetrical discriminator, with two diode circuits having their selectivity curves of the same shape (though tuned to different frequencies), D will increase when all the sideband frequencies are attenuated except the first two ($F_c \pm F_{cn}$). The maximum possible interference with a symmetrical discriminator would occur when the first pair of sideband components fall on the peaks of the discriminator curve, while the rest of the components are completely eliminated. If the circuits of the discriminator are not symmetrical, the interference may become still higher if one sideband is more attenuated than the other. When all the sideband components are attenuated, D_1 and D_2 both approach the same value, so D approaches zero.

It was found that, for a good degree of approximation, when $\rho \geqq 2$ at least two of the sideband components on each side of the carrier have to be taken into consideration. The direct-current component is represented by an infinite series (17) whose terms become so long and complicated that it would be impractical to go any further than the first two terms. Fortunately, for $\rho \geqq 2$ the first term only gives an accuracy within 20 per cent and the second term would bring it within less than 5 per cent. Numerical calculations require an accuracy to at least six figures, which makes the whole procedure long and tedious. So, when valid, the equivalent-frequency-deviation method is easier to use.

Conclusion

As was shown earlier, the interference of two transmitters working on the same channel is caused by the beat frequency between the signals. The carrier of the desired signal becomes frequency modulated by the beat frequency during the process of limiting. This kind of interference cannot be eliminated, or even materially reduced by modifications in the receiver design. The co-channel frequency-modulation interference decreases with decrease of F_{cn}. This effect is partially offset by the increase of the residual amplitude-modulation interference for small F_{cn}. So the only way to reduce the frequency-modulation co-channel interference is to keep the difference between the frequencies of the transmitters as low as practical, while the residual amplitude-modulation interference can be reduced by more effective limiters and accurate tuning.

When the interference is caused by transmitters working on adjacent or alternate channels, the beat frequency is far beyond the audio range. In such a case, interference is due to two factors: First, a direct-current component in the output of the discriminator. This component appears when one or both peaks of the frequency deviation (due to the interfering signal) extend into the nonlinear part of the discriminator curve. As long as the interfering signal is an unmodulated carrier, the direct-current component will remain constant. So, by itself, this component will not produce any audible effect, unless the second factor is present. The second factor is the amplitude variation of the frequency-modulated interfering signal, caused by the steep sides of the receiver selectivity curve. The variation of the amplitude of the interfering signal (with the modulation frequency) causes the direct-current component to vary at the same rate, thus producing an audible interference. So a reduction or elimination of either factor will result in a reduction, or almost complete elimination, of the interference. It must be understood, however, that this is true only as long as the peaks of the interfering signal always remains weaker than the carrier of the desired signal; that is, $\rho > 1$.

The amplitude variation of the interfering signal can be eliminated (or at least considerably reduced) by making the selectivity curve almost flat from 120 to 280 kilocycles and from 320 to 480 kilocycles off resonance (for adjacent and alternate channels, respectively).

The direct-current component at the output of the discriminator can be reduced considerably by the following methods:

(a) By retuning the receiver after a transmitter from an adjacent or alternate channel begins to interfere with the received signal. However, it would not be practical to retune a receiver everytime an interfering signal comes on, or changes in strength, unless it is done automatically. An efficient automatic frequency control would reduce this kind of interference appreciably.

Thus, a receiver with crystal-tuned push buttons is at a disadvantage. The amount of detuning necessary to reduce the direct-current component to zero was plotted against ρ for adjacent-channel interference (Fig. 5, curve D).

(b) Theoretically, the direct-current component can be completely eliminated by making the linear part of the discriminator curve infinitely long. The direct-current component would be reduced appreciably if the linear part was extended, at least as far as $2F_{cn}$ on each side of the carrier. This would mean a reduction of interference at the expense of sensitivity. Many compromises and combinations of the methods described are possible.

Appendix

The compo ite signal at the output of the limiter can be represented by the expression

$$e = A \sin \left\{ \omega_c t + \beta \cos pt + \tan^{-1} \frac{\sin [\omega_{cn} t - \phi(t) + \beta \cos pt]}{\rho + \cos [\omega_{cn} t - \phi(t) + \beta \cos pt]} \right\}. \quad (10)$$

When neither signal is modulated, (1) becomes

$$e = A \sin \left[\omega_c t + \tan^{-1} \frac{\sin \omega_{cn} t}{\rho + \cos \omega_{cn} t} \right].$$

Let

$$\tan^{-1} \frac{\sin \omega_{cn} t}{\rho + \cos \omega_{cn} t} = \alpha. \quad (11)$$

$$e = A \sin (\omega_c t + \alpha) = A(\sin \omega_c t \cos \alpha + \cos \omega_c t \sin \alpha) \quad (12)$$

where $\sin \alpha = \sin \omega_{cn} t (1+\rho^2+2\rho \cos \omega_{cn} t)^{-1/2}$ and $\cos \alpha = (\rho + \cos \omega_{cn} t)(1+\rho^2+2\rho \cos \omega_{cn} t)^{-1/2}$. Substituting $(1+\rho^2) = g$, $2\rho/g = g$, and expanding $(1+\rho^2+2\rho \cos \omega_{cn} t)^{-1/2}$ in terms of the harmonics $\cos n\, \omega_{cn} t$,

$$\sin \alpha = \frac{\sin \omega_{cn} t}{q^{1/2}} \sum_{r=0}^{r=\infty} (-1)^r h_r \cos r\omega_{cn} t \quad (13)$$

where

$$\cos \alpha = \frac{\rho + \cos \omega_{cn} t}{q^{1/2}} \sum_{r=0}^{r=\infty} (-1)^r h_r \cos r\omega_{cn} t$$

$$h_0 = 1 + \sum_{k=1}^{k=\infty} \frac{(4k-1)! g^{2k}}{2^{2(3k-1)} k! (2k)! (k-1)!} \quad (13a)$$

and

$$h_r = \sum_{k=0}^{k=\infty} \frac{(4k+2r-1)! g^{2k+r}}{(2k+r-1)! 2^{(6k+3r-2)} k! (k+r)!}.$$

Substituting (13) and (13a) into (12), we obtain

$$e = a_0 \sin \omega_c t + \sum_{r=1}^{r=\infty} (-1)^r [b_r \sin (\omega_c - r\omega_{cn}) t - a_r \sin (\omega_c + r\omega_{cn}) t] \quad (14)$$

where

$$a_0 = \left(\rho h_0 - \frac{h_1}{2}\right) A \qquad a_3 = (\rho h_3 - h_4) \frac{A}{2}$$

$$a_1 = \left(h_0 - \frac{\rho h_1}{2}\right) A \qquad a_4 = (h_3 - \rho h_4) \frac{A}{2}$$

$$a_2 = (h_1 - \rho h_2) \frac{A}{2} \qquad b_r = (\rho h_r - h_{r+1}) \frac{A}{2}$$

$$r = 1, 2, 3, \cdots.$$

For reasonable accuracy a fundamental and four sideband components (two pair) would be sufficient. When applied to a diode circuit, this expression will have different coefficients due to the selectivity of the circuit. Equation (14) will become

$$e = \alpha_0 \sin \omega_c t + \sum_{r=1}^{r=\infty} (-1)^r [\beta_r \sin (\omega_c - r\omega_{cn}) t - \alpha_r \sin (\omega_c + r\omega_{cn}) t]. \quad (15)$$

The output of the diode (linear detector) is[3]

$$\begin{aligned} E = [&\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \beta_1^2 + \beta^2 \\ &+ 2(\alpha_0\alpha_1 + \alpha_0\beta_1 + \alpha_1\alpha_2 + \beta_1\beta_2) \cos \omega_{cn} t \\ &+ 2(\alpha_0\alpha_2 + \alpha_0\beta_2 + \alpha_1\beta_1) \cos 2\omega_{cn} t \\ &+ 2(\alpha_1\beta_2 + \beta_1\alpha_2) \cos 3\omega_{cn} t + 2\alpha_2\beta_2 \cos 4\omega_{cn} t]^{1/2}. \quad (16) \end{aligned}$$

Let

$$\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \beta_1^2 + \beta_2^2 = \psi$$

and the rest of the expression under the radical $= \epsilon$.

Then

$$E = (\psi + \epsilon)^{1/2} = \psi^{1/2} \left(1 + \frac{\epsilon}{\psi}\right)^{1/2}$$

$$= E = \psi^{1/2} \left[1 + \frac{\epsilon}{2\psi} - \frac{1}{2 \cdot 4}\left(\frac{\epsilon}{\psi}\right)^2 + \frac{1 \cdot 1 \cdot 3}{2 \cdot 4 \cdot 6}\left(\frac{\epsilon}{\psi}\right)^3 - \cdots\right]. \quad (17)$$

$\psi^{1/2}$ is the first approximation of direct-current component $= D$. The odd powers of (ϵ/ψ) do not contain constant term and, as we are interested only in the direct-current component, can be neglected. The constant part of the second term is:

$$\begin{aligned} \gamma = \frac{2}{8\psi^2} [&(\alpha_0\alpha_1 + \alpha_0\beta_1 + \alpha_1\alpha_2 + \beta_1\beta_2)^2 \\ &+ (\alpha_0\alpha_2 + \alpha_0\beta_2 + \alpha_1\beta_1)^2 + (\alpha_1\beta_2 + \beta_1\alpha_2)^2 \\ &+ (\alpha_2\beta_2)^2]. \quad (18) \end{aligned}$$

For $\rho > 2$ the series for D converges rapidly and the first two terms give an accuracy sufficient for all practical purposes.

The direct-current component of a diode is $D_1 \cong \psi_1^{1/2} + \gamma$.

[3] W. L. Everitt, "Communication Engineering," chap. 13, pp. 407; McGraw-Hill Book Co., New York, N. Y.